Interactive Supplementary Lesson Two

General Instructions: Choose the correct answer for the following questions, which are designed for your interactive supplementary practice. The questions, correct answers, and distractive choices are linked. Attempt to answer them, and then check your response by clicking on the alternatives, which will lead you to feedback. You may attempt each question three times until you get it correct. Every time, you get it wrong, you should click on 'Try Again' to return to the same question.

1. Given arithmetic sequence with $A_2 = 3$ and $A_5 = 24$, what is the value of A_7 and A_1?
   A. 77 and 7
   B. 7 and -4
   C. 66 and -4
   D. 66 and 7
2. Given that the 1st term of a geometric sequence is 1, and its common ratio is 3, find the 2nd, 3rd, 4th and 5th term.
   A. {9, 27, 81, 243}
   B. {3, 27, 81, 243}
   C. {3, 9, 27, 81}
   D. $\{^1/_9, ^1/_{27}, ^1/_{81}, ^1/_{243}\}$
3. Given that A_4=15 and A_8=39, then find A_n and d.
   A. $A_n = 6_n + 9$ and $d = 3$
   B. $A_n = 6_n - 9$ and $d = -3$
   C. $A_n = -3 + 6_n - 9$ and $d = -3$
   D. $A_n = 6_n - 9$ and $d = -3$
4. Find G_n when 1st term of a geometric sequence is 3, and its common ratio is 3.
   A. $G_n = 3^n$
   B. $G_n = 3^{n+1}$
   C. $G_n = 3^{n-1}$
   D. $G_n = 3^{2n}$
5. The evaluation of the sigma notation $\sum_{k=1}^{8}(k^3 + 2k^2 - 3k + 5)$ is:
   A. 315
   B. 590
   C. 1015
   D. 1636

IncorrectAnswertoQ1. Try Again.

IncorrectAnswertoQ2. Try Again.

IncorrectAnswertoQ3. Try Again.

IncorrectAnswertoQ4. Try Again.

IncorrectAnswertoQ5. Try Again.

Feedback

Good job. Have a look at the explanation for #Q1 below if you like and then go to the next question.
To find the general term A_n and the first term A_1 of the arithmetic sequence, we can use the information:
Given:
$A_2 = 3$
$A_5 = 24$
Step 1: Express the General Term
The general formula for the n_{th} term of an arithmetic sequence is given by:
$A_n = A_1 + (n-1)\, d$
Where (d) is the common difference.
Step 2: Set Up Equations
From the given information:
1. For A_2:

$A_2 = A_1 + (2 - 1)d \Rightarrow A_2 = A_1 + d$; thus, we have: $(A_1 + d = 3$ ~~~ (1)

2. For A_5:

$A_5 = A_1 + (5 - 1)d \Rightarrow A_5 = A_1 + 4d$; thus, we have: $A_1 + 4d = 24$~~~ (2)

Step 3: Solve the System of Equations
Now we have a system of equations:
1. $A_1 + d = 3$
2. $A_5 + 4d = 24$

We can solve these equations step by step.
From equation (1), we can express, $d = 3 - A_1$
Substituting d in equation (2): $A_1 + 4(3 - A_1) = 24 \Rightarrow A_1 + 12 - 4A_1 = 24$
$= -3A_1 + 12 = 24 = -3A_1 = 24 - 12 = -3A_1 = 12$
$\therefore \underline{A_1 = -4}$
Step 4: Find d

Now substituting A_1 back to find d: $d = 3 - A_1 = 3 - (-4) = 3 + 4 = 7$
Step 5: General Term A_n

Now we can find the general term $\mathbf{A_n}$: $A_n = A_1 + (n-1)d = -4 + (n-1)7$
$A_n = -4 + 7n - 7 \Rightarrow A_7 = \underline{7(7) - 11 = 66}$

In Summary
- $A_1 = -4$
- A7 = 66

Feedback

Good job. Have a look at the explanation for #Q2 below if you like and then go to the next question.

Solution:

In the given geometric sequence, the first term A_1 is 1, and the common ratio r is 3.

General Formula

The n-th term of a geometric sequence can be expressed as: $A_n = A_1 \cdot r^{(n-1)}$

Finding the Terms

2nd Term *A2*:

$$A_2 = 1 \cdot 3^{(2-1)} = 1 \cdot (3) = 3$$

3rd Term *A3*:

$$A_3 = 1 \cdot 3^{(3-1)} = 1 \cdot 3^2 = 9$$

4th Term *A4*:

$$A_4 = 1 \cdot 3^{(4-1)} = 1 \cdot 3^3 = 27$$

5th Term *A5*:

$$A_5 = 1 \cdot 3^{(5-1)} = 1 \cdot 3^4 = 81$$

∴ Summary of Results are:

2nd Term: $A2=3$

3rd Term: $A3=9$

4th Term: $A4=27$

5th Term: $A5=81$

Feedback

Good job. Have a look at the explanation for #Q3 below if you like and then go to the next question.

Solution:

1) Using the formula: $A_4 = A_1 + 3d = 15$ ~~~~~ (1)

$A_8 = A_1 + 7d = 39$ ~~~~~ (2)

2. From (1), solve for A_1: $A_1 + 3d = 15 \Rightarrow A_1 = 15 - 3d$ ~~~~~ (3)
3. Substitute (3) into (2): $A_8 = (15 - 3d) + 7d = 39$

Simplifying gives: $\Rightarrow 15 + 4d = 39$

$\Rightarrow 4d = 24 \Rightarrow d = 6$

4. Substitute *d* back into (3): $A_1 = 15 - 3(6) = 15 - 18 = -3 A_1$

$= 15 - 3(6) = 15 - 18 = -3$

5. The general formula is:

$A_n = -3 + (n-1) \cdot 6 = -3 + 6_n - 6 = 6_n - 9$

$\therefore$ $\underline{A_n}$ = $\underline{6_n - 9}$ and $\underline{d = -3}$

Feedback

Good job. Have a look at the explanation for #Q4 below if you like and then go to the next question.

To find the n_{th} term, G_n of a geometric sequence, we can use the formula:

$$G_n = G_1.r^{(n-1)}$$

Where:

- $-G_1$ is the first term of the geometric sequence,
- R is the common ratio,
- n is the term number.

Given:

- $G_1 = 3$
- $r = 3$

Now, we can substitute the values into the formula:

$$G_n = 3.3^{(n-1)}$$

This can be simplified to:

$$G_n = 3^1 . 3^{(n-1)} = 3^n$$

∴ the n-th term of the geometric sequence is: $G_n = 3^n$

Feedback

Good job. Have a look at the explanation for #Q5 below if you like.

To evaluate the sum $\sum_{k=1}^{8}(k^3 + 2k^2 - 3k + 5)$, we need to calculate each term in the summation as *k* varies from 1 to 8 and then sum the results. Thus:

1. For k = 1: $1^3 + 2(1^2) - 3(1) + 5 = 1 + 2 - 3 + 5 = 5$
2. For k = 2: $2^3 + 2(2^2) - 3(2) + 5 = 8 + 6 - 3 + 5 = 15$
3. For k = 3: $3^3 + 2(3^2) - 3(3) + 5 = 27 + 18 - 9 + 5 = 41$
4. For k = 4: $4^3 + 2(4^2) - 3(4) + 5 = 64 + 32 - 12 + 5 = 89$
5. For k = 5: $5^3 + 2(5^2) - 3(5) + 5 = 125 + 50 - 15 + 5 = 165$
6. For k = 6: $6^3 + 2(6^2) - 3(6) + 5 = 216 + 72 - 18 + 5 = 275$
7. For k = 7: $7^3 + 2(7^2) - 3(7) + 5 = 343 + 98 - 21 + 5 = 425$
8. For k = 8: $8^3 + 2(8^2) - 3(8) + 5 = 512 + 128 - 24 + 5 = 621$

Sum the Results

Now, we need to add up all the results from k=1 to k=8:

$\therefore \sum_{k=1}^{8}(k^3 + 2k^2 - 3k + 5)$ is:

$$5 + 15 + 41 + 89 + 165 + 275 + 425 + 621 = 1636$$