Interactive Supplementary Lesson Three

General Instructions: Choose the correct answer for the following questions, which are designed for your interactive supplementary practice. The questions, correct answers, and distractive choices are linked. Attempt to answer them, and then check your response by clicking on the alternatives, which will lead you to feedback. You may attempt each question three times until you get it correct. Every time, you get it wrong, you should click on 'Try Again' to return to the same question.

1. Compute the average rate of change of $f(x) = x^3 - 3x$ over the interval $1 \leq x \leq 6$.
   A. 85
   B. 40
   C. 33
   D. 28.3
2. The population of a certain bacterial culture in millions as a function of time in hours is given by $P(t) = 50t^2 + 20$. Then what is the instantaneous rate of growth of the population when (t = 2) hours?
   A. 200 million bacteria per hour
   B. 220 million bacteria per hour
   C. 100 million bacteria per hour
   D. 70 million bacteria per hour
3. Given the functions $h(x) = 4x^3 - 5x$ and $k(x) = 2x^2 + 3x + 1$, then find $h'(x) + k'(x)$.
   A. $4x3^2 + 2x^2 - 2x + 1$
   B. $4x3^2 \;\; 2x^2 + 8x + 1$
   C. $12x^2 + 4x + 2$
   D. $12x^2 + 4x - 2$
4. Find the derivative of the function $f(x) = \sqrt{(2x^4 - 5x + 1)^5}$.
   A. $(8x^3 - 5)\sqrt{(x^4 - 6}$
   B. $(8x^3 - 5)\sqrt{(2x^4 - 5x + 1)^3}$
   C. $\frac{5}{2}(2x^4 - 5 + 1)^{1/2}\,(8x^3 - 5)$
   D. $\frac{5}{2}(2x^4 - 5x+1)^{3/2}.\,(8x^3 - 5)$
5. Find the absolute maximum and minimum values of the function $f(x) = x^2 + 2x$ on [-3, 1].
   A. 3 and -1
   B. 3 and 1
   C. 1 and -3
   D. -1 and -3

IncorrectAnsToQ1. Try Again.

IncorrectAnsToQ2. Try Again.

IncorrectAnsToQ3. Try Again.

IncorrectAnsToQ4. Try Again.

IncorrectAnsToQ5. Try Again.

Feedback

Good job. Have a look at the explanation for #Q1 below if you like and then go to the next question.

Solution:

$f(x)=x^3-3x$ over the interval $1 \leq x \leq 6$

1. **Calculate $f(1)$ and $f(6)$**:

   $f(1) = 1^3-3(1) = 1-3 = -2$

   $f(6) = 6^3-3(6) = 216-18=198$

2. **Average Rate of Change**:

Average Rate of Change $= \frac{198-(-2)}{6-1} = \frac{198+2}{5} = \frac{200}{5} = 40$

Good job. Have a look at the explanation for #Q2 below if you like and then go to the next question.

Solution:

To find the instantaneous rate of growth of the population given by the function $P(t) = 50t^2 + 20$ at $t = 2$ hours, we will follow these steps:

1. Find the derivative $P'(t)$.
2. Evaluate the derivative at $t = 2$.

Step 1: Find the Derivative

The function is: $P(t) = 50t^2 + 20$

To find the derivative $P'(t)$, we differentiate with respect to (t):

$P'(t) = \frac{d}{dt}(50t^2) + \frac{d}{dt}(20) = 100t + 0 = 100t$

Understanding the Derivative: P'(t) = 100t

Breakdown of the notation:

- **P'(t):** This represents the derivative of the function $P(t)$. It signifies the instantaneous rate of change of the population (P) with respect to time (t).
- **d/dt:** This is the symbol for differentiation. It means take the derivative with respect to t.
- **($50t^2 + 20$):** This is the original function representing the population as a function of time.

Applying the Power Rule:

The power rule is a fundamental rule of differentiation that states:

- **$d/dx(x^n) = nx^{(n-1)}$**

where:

- n is any real number (except for n = -1); x is a variable

In our case, we have:

- $P(t) = 50t^2 + 20$

We can break this down into two terms:

1. **$50t^2$:** Here, n = 2. Applying the power rule, the derivative is:
    - $d/dt(50t^2) = 50 * 2 * t^{(2-1)} = 100t$
2. **20:** This is a constant term. The derivative of a constant is always 0.

Combining the derivatives:

Since the derivative of the sum of functions is the sum of their derivatives, we can combine the derivatives of the two terms:

- $d/dt(50t^2 + 20) = d/dt(50t^2) + d/dt(20)$
- $= 100t + 0$

Therefore:

- $P'(t) = 100t$

Step 2: Evaluate the Derivative at ($t = 2$)

Now we will evaluate $P'(t)$ at $t = 2$:

$P'(2) = 100(2) = 200$

∴ The instantaneous rate of growth of the population when t = 2 hours is **200 million bacteria per hour.**

Good job. Have a look at the explanation for #Q3 below if you like and then go to the next question.

Solution:

Given Functions: $h(x) = 4x^3 - 5x$ and $k(x) = 2x^2 + 3x + 1$
Step 1: Calculate the Derivatives

1. Derivative of $h(x)$

   $H'(x) \frac{d}{dx}(4x^3 - 5x)$ = 12x^2 – 5

Understanding the Derivative

The derivative of a function gives us the rate at which that function changes with respect to its variable. In this case, we are finding the derivative of the function $h(x) = 4x^3 - 5x$.

Finding the Derivative of *h*(x)

To find the derivative $h'(x)$, we use the rules of differentiation:

Power Rule: For any function of the form ax^n, the derivative is $nax^{(n-1)}$.

Linear Terms: The derivative of a constant multiplied by x is simply the constant.

Apply the Rules to *h*(x)

- Derivative of $4x^3$ using the power rule: $3.4\, x^{3-1} = 12x^2$
- Derivative of - $5x$: using the linear term rule: -5

Now we combine the derivatives: $h'(x) = 12x^2 - 5$

2. Derivative of k(x):

   $k'(x) = \frac{d}{dx}(2x^2 + 3x + 1) = 4x + 3$

Step 2: Evaluate the Expression $h'(x) + k'(x)$ \)

To find $h'(x) + k'(x)$: $h'(x) + k'(x) = (12x^2 - 5) + (4x + 3)$

Combine the terms: $h'(x) + k'(x)$ = 12x^2 + 4x - 5 + 3 = $12x^2 + 4x - 2$

Final Result: $h'(x) + k'(x) = 12x^2 + 4x - 2$

Good job. Have a look at the explanation for #Q4 below if you like and then go to the next question.

Let's solve the derivative of the function $f(x) = \sqrt{(2x^4 - 5x + 1)^5}$

Step 1: Rewrite the Function

So, we can rewrite the function as: $f(x)$ =$(2x^4 - 5x + 1)^{5/2}$.

Step 2: Identify Outer and Inner Functions

Outer function: $g(u)$ = $u^{5/2}$ where u = 2 x^4 - 5x + 1

Inner function: $h(x)$ = 2 x^4 - 5x + 1

Step 3: Differentiate the Outer Function

Differentiate the outer function $g(u) \Rightarrow g'(u)$ = $\frac{5}{2}u^{3/2}$

Step 4: Differentiate the Inner Function

Differentiate the inner function $h(x) \Rightarrow h'(x))$ = $\frac{d}{dx}$(2x^4 - 5x +1) = 8x^3 - 5.

Step 5: Apply the Chain Rule

Using the chain rule: $f(x)$ = $g'(h(x)).h'(x)$.

Substituting back in, we have: $f'(x)$ = $\frac{5}{2}(2x^4$ - $5x + 1)^{3/2}$. $(8x^3$ - $5)$

Final Result of the derivative:

$\boldsymbol{f(x) = \sqrt{(2x^4 - 5x + 1)^5}}$ is: $\boldsymbol{f'(x) = \frac{5}{2}(2x^4 - 5x + 1)^{3/2}\ (8x^3 - 5)}$.

Convert to Square Root Form

Notice that $\boldsymbol{(2x^4 - 5x + 1)^{3/2}}$ can be written in terms of square roots:

$\boldsymbol{(2x^4 - 5x + 1)^{3/2}}$ = $\boldsymbol{\sqrt{(2x^4 - 5x + 1)^3}}$

Substitute Back into the Derivative

Then, our derivative in terms of square root will be:

$\boldsymbol{f'(x)}$ = $\boldsymbol{\frac{5}{2}\sqrt{(2x^4 - 5x + 1)^3}\ (8x^3 - 5)}$.

Final Result in Square Root Form

Thus, the derivative of the function $\boldsymbol{f(x) = \sqrt{(2x^4 - 5x + 1)^5}}$ expressed in terms of square roots is:

$\boldsymbol{f'(x)}$ = $\boldsymbol{\frac{5}{2}(8x3\ -\ 5)\sqrt{(2x^4 - 5x + 1)^3}}$

Good job. Have a look at the explanation for #Q5 below if you like.

Find the absolute maximum and minimum values of the function $f(x) = x^2 + 2x$ on [-3, 1].

Solution:

To determine the absolute extrema of the function $f(x) = x^2 + 2x$ on the interval [-3, 1], we will follow the same steps:

1. Find the derivative of the function $f(x)$.
2. Solve $f(x) = 0$ to determine the critical points.
3. Evaluate the function at the critical points and the endpoints of the interval to find the absolute maximum and minimum values.

Step 1: Find the derivative of $f(x)$.

The given function is: $f(x) = x^2 + 2x$

The derivative $f'(x) = \frac{d}{dx}(x^2 + 2x) = 2x + 2$

Step 2: Solve $f'(x) = 0$

Set the derivative equal to 0 and solve for (x): $2x + 2 = 0$

$2x = -2$

$x = -1$

So, the critical point is $x = -1$.

Step 3: Evaluate the function at the endpoints and the critical point

Evaluate $f(x)$ at $x = -3$, $x = 1$, and $x = -1$:

1. At $x = -3$: $f(3)$) = $(-3)^2 + 2(-3) = 9 - 6 = 3$
2. At $x = 1$: $f(1)$) = $(1)^2 + 2(1) = 1 + 2 = 3$
3. At $x = -1$: $f(-1) = (-1)^2 + 2(-1) = 1 - 2 = -1$

∴

1. The absolute maximum value of the function $f(x) = x^2 + 2x$ on the interval [-3, 1] is 3 (occurring at $x = -3$ and $x = 1$.
2. The absolute minimum value of the function $f(x) = x^2 + 2x$ on the interval [-3, 1] is -1(occurring at $x = -1$.